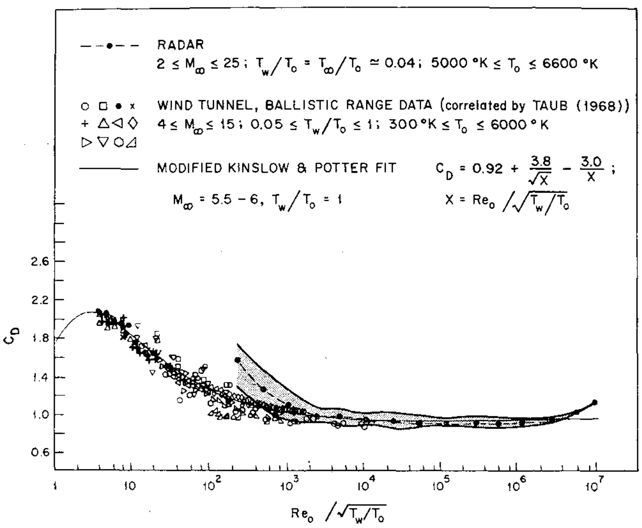

Fig. 2 Radar and laboratory sphere drag correlation.

behind the shock, and variability from the mean model atmosphere contribute to measurement errors. The effect of these errors decreases as the drag deceleration of the spheres increases; therefore, the results presented here are limited to the slip and continuum flow regimes.

III. Results

Average C_D for the sphere experiments and the envelopes about the measurements at selected points are shown in Fig. 1. The results are presented as a function of Reynolds number, $Re = Re_\infty/M_\infty$ where $Re_\infty = \rho_\infty v_\infty D/\mu_\infty$, μ_∞ is the coefficient of viscosity, the subscript ∞ denotes freestream conditions, and M_∞ is the Mach number also shown in Fig. 1 for one of the sphere flights. The mean drag values are based on 7 sphere flights in the range $Re \leq 5 \times 10^4$, 5 observations at $Re = 5 \times 10^5$, and 2 measurements at $Re > 5 \times 10^5$. The variability of the experiments at $Re \leq 2 \times 10^2$ is characterized by a coefficient of variation about 10%. At $Re \geq 5 \times 10^2$, the coefficient of variation is less than 5% and pertains largely to air density deviations from the mean model.

To allow the comparison of the radar results with the laboratory data correlated by Taub[1] several assumptions must be made. The normalization of Re_∞ by M_∞ characterizes Re for conditions behind the normal shock, hence stagnation, assuming temperature equality for both conditions. To modify Re with respect to $(T_w/T_o)^{1/2}$ (subscripts w and o denoting wall and stagnation conditions, respectively) T_o is obtained from gas dynamic charts for equilibrium air (e.g., Feldman[3]) and T_w is assumed equal to T_∞. The latter assumption is particularly valid for $Re < 10^3$ as suggested by theoretical computations[4] for spheres of various materials; at $Re > 10^3$ departure of T_w from T_∞ within cold wall conditions would only shift the drag results along the continuum value.

The comparison between laboratory and field measurements of C_D expressed as a function of $X = Re_o/(T_w/T_o)^{1/2}$ is provided in Fig. 2 together with the modified Kinslow and Potter[5] fit. The radar data at $X < 2.5 \times 10^2$ are regarded with little confidence as previously noted and are not shown. In view of the assumptions governing the radar results and the variability of the observations due to the combined effects described earlier, the discrepancy between the ground and flight data during slip flow ($2.5 \times 10^2 < X < 10^3$) is reasonably well understood. The agreement between the two sources of measurements during continuum flow ($X > 10^3$) is observed to be excellent. The rising trend in C_D at $X \geq 5 \times 10^6$ reflects the drag variability when the spheres approach transonic velocities as has been observed by Hodges.[6] From this preliminary analysis, it may be concluded that in the slip and continuum flow regimes the wind-tunnel and ballistic range data are confirmed satisfactorily by radar measurements.

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Pressure Distribution in Inviscid Transonic Flow past Axisymmetric Bodies ($M_\infty = 1$)

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HOSOKAWA'S[1] technique of solving the nonlinear transonic flow equation has been applied to determine the pressure distributions on circular-arc bodies of revolution for $M_\infty = 1$. The results are compared with Spreiter's local linearization method[2] and also experimental results.[6,7] On comparison, it is observed that the present method predicts a shock wave near the trailing edge of the body which is confirmed by the experimental results. Spreiter's method, on the other hand, does not predict a shock wave. The smearing of the pressure distribution in the neighborhood of the shock wave due to the shock boundary layer interaction, as shown by the experimental results, can not be predicted by the purely inviscid theory.

The nonlinear equation in the case of axisymmetric flow is

$$(1 - M_\infty{}^2)\Phi_{xx} + \Phi_{rr} + (1/r)\Phi_r = (\gamma + 1)M_\infty{}^2\Phi_x\Phi_{xx} \quad (1)$$

where Φ is the perturbation velocity potential divided by U_∞. U_∞ and M_∞ are the freestream velocity and Mach number, respectively, γ is the ratio of specific heats, and x and r are cylindrical coordinates. Φ is defined such that

$$\Phi = \varphi + g \quad (2)$$

where φ, the linearized potential solution, is given by Maeder[3] as

$$\varphi(\hat{x},r) = 2t^2x[(1 - 3x + 2x^2)\log(e^C K\gamma^2/4x) + (11/3)x^2 - (9/2)x + 1] \quad (3)$$

C being Euler's constant.

The nonlinear correction term g, due to Hosokawa, obtained by an order of magnitude analysis due to Guderley,[4] is given

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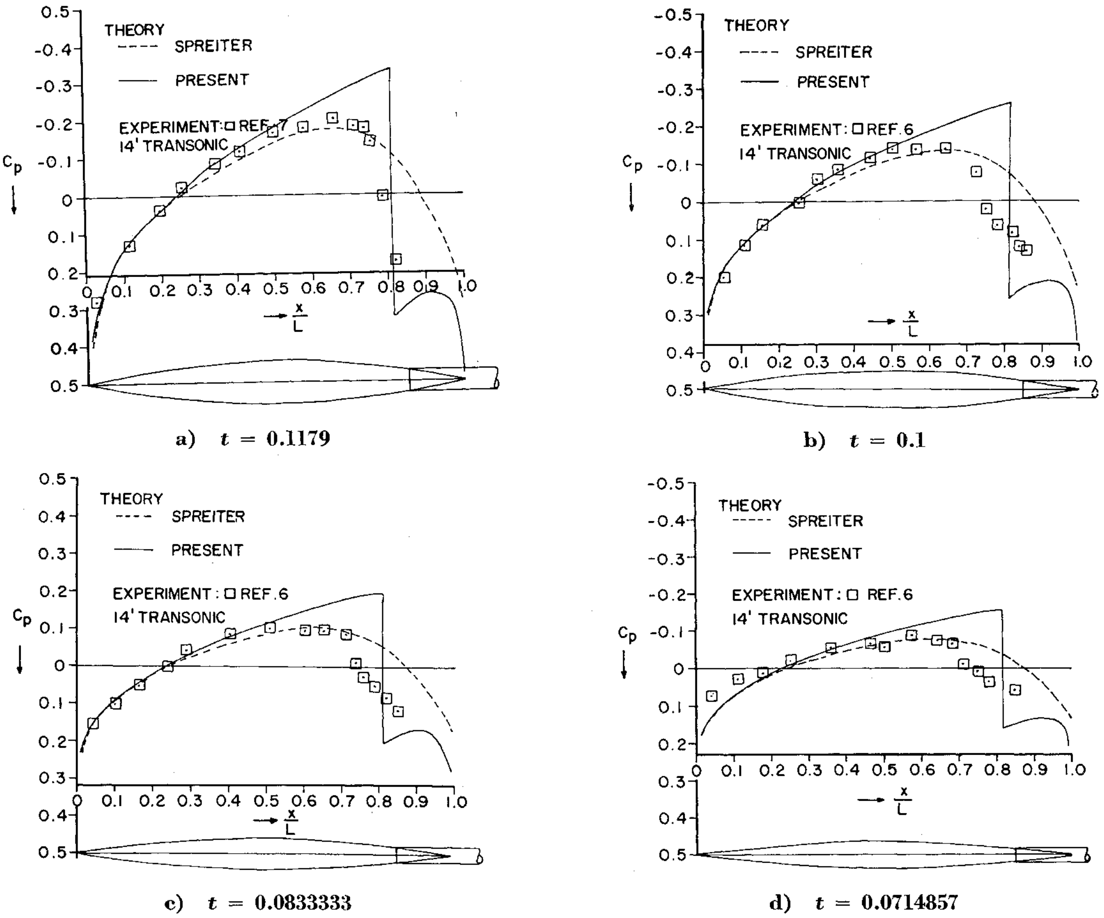

Fig. 1 Pressure coefficients on a circular-arc body of revolution.

by

$$g_x = -\left\{\varphi_x - \frac{1 - M_\infty{}^2}{(\gamma + 1)M_\infty{}^2}\right\} \pm \left[\left\{\varphi_x - \frac{1 - M_\infty{}^2}{(\gamma + 1)M_\infty{}^2}\right\}^2 - 2\int_c^x \left[\varphi_{xx} - \frac{K}{(\gamma + 1)M_\infty{}^2}\right]\varphi_x dx\right]^{\frac{1}{2}} \quad (4)$$

in which the double sign should be taken according as

$$\varphi_x - (1 - M_\infty{}^2)/(\gamma + 1)M_\infty{}^2 \gtrless 0 \quad (5)$$

The sonic point is the same for the original linearized flow, as well as the corrected flow, as $g_x(c^*) = 0$ where $x = c^*$ satisfies $\varphi_x = 0$ which gives $c = c^*$. The parameters c^* and K are determined by simultaneous solution of the equations

$$K = (\gamma + 1)\varphi_{xx}(c^*) \quad (6)$$

and

$$\varphi_x(c^*) = 0 \quad (7)$$

using for example, the Newton-Raphson method.

Equation (7) has another root c^{**} in the decelerated flow region, which gives the shock-wave position. The shock-wave position occurs near the trailing edge when $M_\infty = 1$.

The pressure coefficient C_p is given by

$$C_p = -2\Phi_x - (\varphi_r)^2 \quad (8)$$

where

$$\Phi_x = \pm\{2K/(\gamma + 1)[\varphi(x) - \varphi(c^*)]\}^{1/2} \quad (9)$$

and

$$\varphi_r = 2t(1 - 2x) \quad (10)$$

The values of C_p so obtained are plotted against X/L (for bodies with $t = 0.1179$, 0.1000, 0.0714857, and 0.0833333), where L is the length of the body, and these values are compared with those of Spreiter and experimental results (Figs. 1a–1d).

In all these cases it can be observed that the proposed method compares favorably with Spreiter's as well as experimental results up to $X/L \approx 0.6$. There is an improvement near the trailing edge region, where the existence of the shock is supported by the experimental results. Downstream of the shock, near the trailing edge, there is first a decrease in pressure and afterwards an increase. This fact is observed from the graphs, where there is a kink in the subsonic flow region. Such a behavior has been observed by Crown[5] for thick airfoils as well.

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